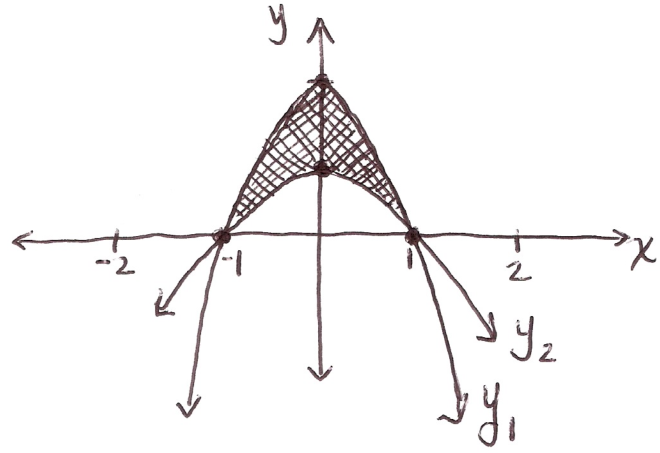
More Area Between Curves Solutions

4-131)

a) $y_1 = -2(x^2 - 1)$

$y_2 = -x^2 + 1$



Intersection: $y_1 = y_2$

$-2(x^2-1) = -x^2+1$

$-2x^2+2 = -x^2+1$

$-x^2+1 = 0$

$-1(x^2-1) = 0$

$-1(x+1)(x-1) = 0$

$x+1=0 \;;\; x-1=0$

$x=-1 \;;\; x=1$
$(a=-1) \;\;\; (b=1)$

Area $= \int_a^b (y_1 - y_2)\,dx$

$= \int_{-1}^{1} \left[-2(x^2-1) - (-x^2+1)\right] dx$

$= \int_{-1}^{1} (-2x^2 + 2 + x^2 - 1)\,dx$

$= \int_{-1}^{1} (-x^2+1)\,dx$

$= \left(-\frac{1}{3}x^3 + x\right)\Big|_{-1}^{1}$

$= \left(-\frac{1}{3}(1^3)+1\right) - \left(-\frac{1}{3}(-1)^3 + (-1)\right)$

$= \left(-\frac{1}{3}+1\right) - \left(\frac{1}{3}-1\right)$

$= -\frac{1}{3} + 1 - \frac{1}{3} + 1$

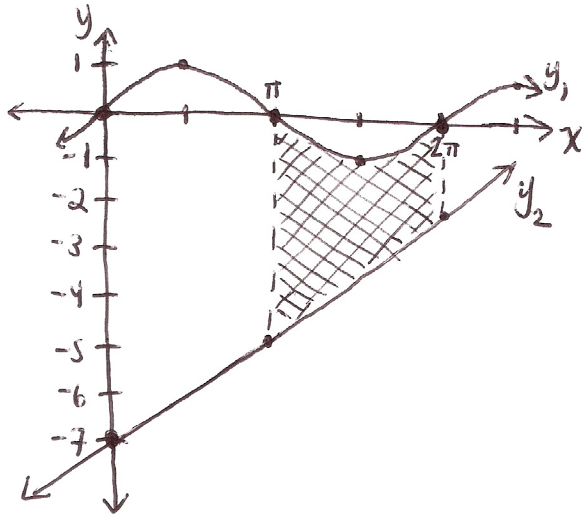
$= -\frac{2}{3} + 2$

$= \frac{4}{3}$ units^2

4-131 continued)

b) $y_1 = \sin x$

$y_2 = \frac{3}{4}x - 7$; $\pi \le x \le 2\pi$



$$\text{Area} = \int_{\pi}^{2\pi} (y_2 - y_1)\,dx$$

$$= \int_{\pi}^{2\pi} \left(\left(\frac{3}{4}x - 7\right) - \sin x\right)dx$$

$$= \int_{\pi}^{2\pi} \left(\frac{3}{4}x - 7 - \sin x\right)dx$$

$$= \left(\frac{3}{8}x^2 - 7x + \cos x\right)\Big|_{\pi}^{2\pi}$$

$$= \left[\frac{3}{8}(2\pi)^2 - 7(2\pi) + \cos(2\pi)\right] - \left[\frac{3}{8}\pi^2 - 7\pi + \cos(\pi)\right]$$

$$= \left(\frac{12\pi^2}{8} - 14\pi + 1\right) - \left(\frac{3\pi^2}{8} - 7\pi - 1\right)$$

$$= \frac{12\pi^2 - 3\pi^2}{8} - 14\pi + 7\pi + 1 + 1$$

$$= \frac{9\pi^2}{8} - 7\pi + 2 \text{ units}^2$$

or ≈ -8.887843624 units^2

4-13a) Since $f(x) > g(x)$ on $x \in (a, b)$, if we calculated $\int_a^b (g(x) - f(x))dx$ instead of $\int_a^b (f(x) - g(x))dx$, the value for the area / definite integral would become negative since we would be subtracting a larger area from a smaller one.

This makes sense since the following relationship is true:

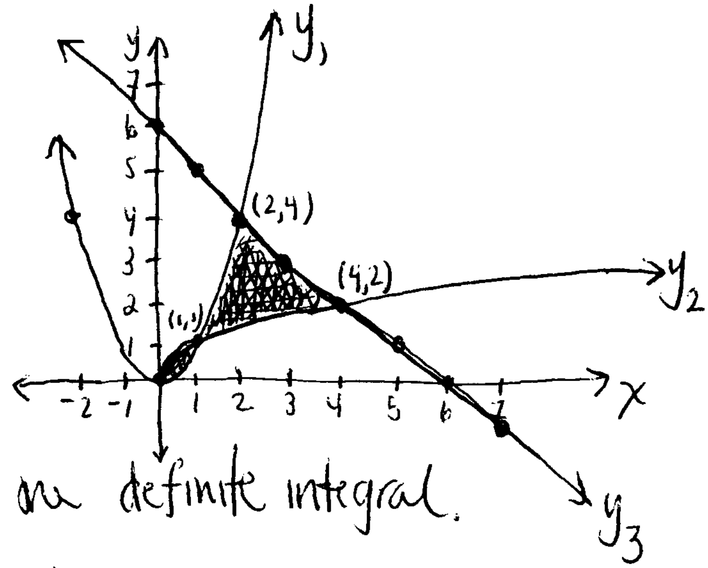
$$\int_a^b (g(x) - f(x))dx = \int_a^b -(g(x) - f(x))dx$$

$$= \int_a^b (-g(x) + f(x))dx$$

$$= \int_a^b (f(x) - g(x))dx$$

4-133) $y_1 = x^2, \quad y_2 = \sqrt{x}, \quad y_3 = -x + 6$

a) Since the three curves do not all share the same upper and lower boundaries with respect to the regions of which we are finding the areas, we cannot evaluate with only one definite integral.



b) $$\text{Area} = \int_a^b (y_2 - y_1)\,dx + \int_b^c (y_1 - y_2)\,dx + \int_c^d (y_3 - y_2)\,dx$$

$$= \int_0^1 (\sqrt{x} - x^2)\,dx + \int_1^2 (x^2 - \sqrt{x})\,dx + \int_2^4 ((-x+6) - \sqrt{x})\,dx$$

$$= \int_0^1 (x^{1/2} - x^2)\,dx + \int_1^2 (x^2 - x^{1/2})\,dx + \int_2^4 (6 - x - x^{1/2})\,dx$$

$$= \left[\left(\tfrac{2}{3}x^{3/2} - \tfrac{1}{3}x^3\right)\Big|_0^1\right] + \left[\left(\tfrac{1}{3}x^3 - \tfrac{2}{3}x^{3/2}\right)\Big|_1^2\right] + \left[\left(6x - \tfrac{1}{2}x^2 - \tfrac{2}{3}x^{3/2}\right)\Big|_2^4\right]$$

$$= \left[\left(\tfrac{2}{3} - \tfrac{1}{3}\right) - 0\right] + \left[\left(\tfrac{8}{3} - \tfrac{4\sqrt{2}}{3}\right) - \left(\tfrac{1}{3} - \tfrac{2}{3}\right)\right] + \left[\left(24 - 8 - \tfrac{16}{3}\right) - \left(12 - 2 - \tfrac{4\sqrt{2}}{3}\right)\right]$$

$$= \left(\tfrac{1}{3} - 0\right) + \left(\tfrac{9}{3} - \tfrac{4\sqrt{2}}{3}\right) + \left[\left(16 - \tfrac{16}{3}\right) - \left(10 - \tfrac{4\sqrt{2}}{3}\right)\right]$$

$$= \tfrac{1}{3} + \tfrac{9}{3} - \tfrac{4\sqrt{2}}{3} + \left(6 - \tfrac{16}{3} + \tfrac{4\sqrt{2}}{3}\right)$$

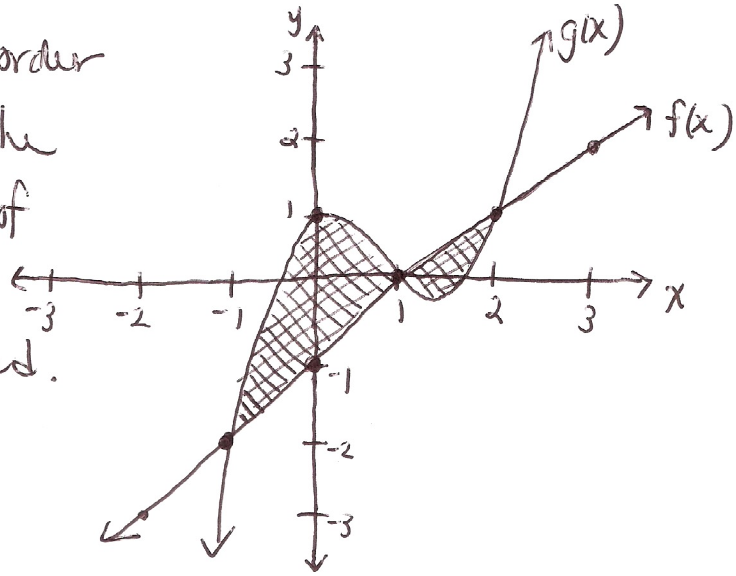
$$= -\tfrac{6}{3} + 6$$

$$= -2 + 6$$

$$= \underline{\underline{4 \text{ units}^2}}$$

4-134) $f(x) = x - 1$; $g(x) = x^3 - 2x^2 + 1$

a) Since we are reversing the order in which we are subtracting the functions to find the areas of the two respective regions, two definite integrals are needed.



b) Area $= \int_{-1}^{1} (g(x) - f(x))dx + \int_{1}^{2} (f(x) - g(x))dx$

$= \int_{-1}^{1} (x^3 - 2x^2 + 1 - (x-1))dx + \int_{1}^{2} (x - 1 - (x^3 - 2x^2 + 1))dx$

$= \int_{-1}^{1} (x^3 - 2x^2 - x + 2)dx + \int_{1}^{2} (-x^3 + 2x^2 + x - 2)dx$

$= \left(\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + 2x\right)\Big|_{-1}^{1} + \left(-\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 - 2x\right)\Big|_{1}^{2}$

$= \left(\left[\frac{1}{4} - \frac{2}{3} - \frac{1}{2} + 2\right] - \left[\frac{1}{4} + \frac{2}{3} - \frac{1}{2} - 2\right]\right) + \left(\left[-4 + \frac{16}{3} + 2 - 4\right] - \left[-\frac{1}{4} + \frac{2}{3} + \frac{1}{2} - 2\right]\right)$

$= \left(-\frac{4}{3} + 4\right) + \left(-4 + \frac{14}{3} + \frac{1}{4} - \frac{1}{2}\right)$

$= \frac{-4 + 12 - 12 + 14}{3} + \frac{1 - 2}{4}$

$= \frac{10}{3} - \frac{1}{4}$

$= \frac{40 - 3}{12}$

$= \frac{37}{12}$ units^2

4-135)

a) $\int_0^5 (|x-2|+3)\,dx = \int_0^2 (-(x-2)+3)\,dx + \int_2^5 ((x-2)+3)\,dx$

$= \int_0^2 (-x+2+3)\,dx + \int_2^5 (x-2+3)\,dx$

$= \int_0^2 (-x+5)\,dx + \int_2^5 (x+1)\,dx$

$= \left(-\frac{1}{2}x^2+5x\right)\Big|_0^2 + \left(\frac{1}{2}x^2+x\right)\Big|_2^5$

$= \left[\left(-\frac{1}{2}(2^2)+5(2)\right)-\left(-\frac{1}{2}(0^2)+5(0)\right)\right]+\left[\left(\frac{1}{2}(5^2)+5\right)-\left(\frac{1}{2}(2^2)+2\right)\right]$

$= (-2+10-0)+\left(\frac{25}{2}+5-2-2\right)$

$= 8+\frac{25}{2}+1$

$= 9+\frac{25}{2}$

$= \frac{18+25}{2}$

$= \frac{43}{2}$ units^2 OR $= 21.5$ units^2

Since the absolute value function can be represented in pieces, we can divide the area of integration in two, evaluating two definite integrals for the area of each region.

4-135)
continued

b) $\int \left(\frac{4}{m^3} - 3\cos(m)\right) dm = \int (4m^{-3} - 4\cos(m)) dm$

$= \underline{\underline{-2m^{-2} - 4\sin(m) + C}}$

Integration through antiderivative power rule and primary trigonometric rule.

c) $\int_1^2 x^x \, dx$ = Area under the curve $f(x) = x^x$ bounded by the x-axis over $x \in [1,2]$.

using calc.

$\approx \underline{\underline{2.050446235 \text{ units}^2}}$

Since we cannot find an antiderivative of $f(x) = x^x$ through elementary functions, we can use the integration command on the calculator to approximate the area.

d) $\int \pi^2 dx = \underline{\underline{\pi^2 x + C}}$

Integration through antiderivative power Rule.